Calorimetry in High-Energy Physics
(lecture 1 of 2)

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Topics (lecture 1 of 2)

- introduction to calorimetry
- electromagnetic showers
- hadronic showers
- detector response and compensation

(by far not exhaustive)
Calorimetry

Name inherited from thermodynamics …
Calorimetry

Name inherited from thermodynamics …

\[ Q: \text{does this make sense?} \]
Calorimetry

Name inherited from thermodynamics …

\[ Q: \text{does this make sense?} \]

possibly yes …

it works on O(100\%) energy absorption

energy \rightarrow \text{heat}
Calorimetry

Heat?

100 TeV $\sim 4 \times 10^{-9}$ Cal over tons of material!
Heat?

$100 \text{ TeV} \sim 4 \times 10^{-9} \text{ Cal over tons of material!}$

not measurable!
Calorimetry

Heat?

100 TeV $\sim 4 \times 10^{-9}$ Cal over tons of material!

not measurable!

$\rightarrow$ use some secondary process
Calorimetry

Secondary processes?

What?
Calorimetry

Secondary processes?

What?

Typically:

- Ionisation
- Scintillation light emission
- Cherenkov light emission
Calorimetry

Take care:
Calorimetry

*Take care*: only a very small fraction of the total energy goes into the secondary process …
Calorimetry

*Take care:*

 only a very small fraction of the total energy goes into the secondary process …

*How much?*
Calorimetry

Take care:

only a very small fraction of the total energy goes into the secondary process …

How much?

$$<< 1 : 10^3$$

(even orders of magnitude lower)
Calorimetry

Take care:

only a very small fraction of the total energy goes into the secondary process …

How much?

\[ \ll 1 : 10^3 \]
(even orders of magnitude lower)

… nevertheless:
Take care:

only a very small fraction of the total energy goes into the secondary process …

How much?

$\ll 1 : 10^3$ (even orders of magnitude lower)

… nevertheless:

proportional to the total energy lost!
Calorimeters

massive detectors for both charged and neutral particles → work as well for clusters of particles (i.e. jets)

particles are ~ totally “absorbed”

absorption process known as “shower development”

typically divided into:

1) electromagnetic (“em”) calorimeters
2) hadronic (“had”) calorimeters
Calorimeters

massive detectors for both charged and neutral particles → work as well for clusters of particles (i.e. jets)

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typically divided into:

1) electromagnetic ("em") calorimeters
2) hadronic ("had") calorimeters

last but not least:

(+) provide (local & global) trigger information
(++) provide particle ID capabilities
Calorimeters

Missing energy measurements:

$4\pi$ (em & had) calorimetry coverage

[“hermeticity”]
development driven by em interactions:

→ clean & ~ simple

→ long-range

→ depend on atomic properties

→ atomic number & atomic scale (~10^{-10} m)
hadronic (had) showers

development driven by nuclear interactions:

→ complex & ~ hard

→ short-range

→ depend on nuclear properties

→ density of nuclei & nuclear scale (~$10^{-15}$ m)
atom $\rightarrow$ football field (electron clouds anywhere)

nucleus $\rightarrow$ 1 mm (static) sand grain at field center

well known for about a century ...
atom → football field (electron clouds anywhere)

nucleus → 1 mm (static) sand grain at field center

→ hadrons need to pass within $\sim 10^{-15} \text{m}$ from nuclei to interact

→ detectors (dimensions, materials) and performance quite different
Cascade of \((e^+, e^-, \gamma) \rightarrow \) stochastic process w/ thousands particles

pair production, bremsstrahlung & ionisation
$X_0$ : longitudinal development scale

$$-\langle \frac{dE}{dx} \rangle_{\text{Brems}} = \frac{E}{X_0}$$

$1 X_0$ : when $<1-1/e>$ ($\sim 63.2\%$) of electron energy $\rightarrow$ brems.

$$X_0 = \frac{1433A}{Z(Z + 1)(11.4 - \ln(Z))} \text{ g cm}^{-2}$$

$$X_0 \ [ \text{g/cm}^2 ] \sim Z^{-1}$$
critical energy $\rightarrow E_c$

$E_c$ : when bremsstrahlung takes over ionisation
critical energy $\rightarrow E_c$

$E_c$ : when bremsstrahlung takes over ionisation

![Graph showing $E_c[\text{MeV}] \sim Z^{-1}$]
Molière radius $\rightarrow R_M$

lateral spread $\sim$ driven by multiple scattering

$R_M$ : radius of cylinder containing 90% of shower energy (95% in $2 \times R_M$)

$$R_M = E_s \frac{X_0}{E_c}$$

where :

$$E_s = m_e c^2 \sqrt{4\pi/\alpha} = 21.2 \text{ MeV}$$

$\rightarrow R_M \ [ \text{g/cm}^2 \ ] \sim$ independent of $Z$
compound materials

\[ \frac{1}{X_0} = \sum \frac{w_j}{X_j} \]

where: \( w_j \) = fraction by weight of \( j \)th element

same for \( R_M \):

\[ \frac{1}{R_M} = \frac{1}{E_s} \sum \frac{w_j E_{cj}}{X_j} = \sum \frac{w_j}{R_{Mj}} \]
High-energy photons $\rightarrow$ mean free path (pair production) $\sim \frac{9}{7} X_0$

at first pair production $\rightarrow$ 2 electron showers
em shower:

- Electrons emit photons.
- Photons produce $e^+e^-$ pairs.
1) fractional energy deposition per $X_0$
2) number of e and photons ($E > 1.5$ MeV) crossing planes
electron vs. $\gamma$ initiated showers
... one more parameter

shower maximum (shower depth):
where multiplication process \( \sim \) stops

\[
X = X_0 \frac{\ln(E_0/E_c)}{\ln 2}
\]

\( X \sim 1 / Z, \sim \log(E) \)

shower longitudinal dimension mildly grows as \( \log(E) \)
shower development

longitudinal profiles

lateral profiles

e in copper

10 GeV e−

after shower maximum, lateral spread dominated by isotropic processes (Compton scattering, photoelectric effect)
scaling violations

As well due to low-energy phenomena (Compton scattering, photoelectric effect) dominating after shower maximum
energy response

total shower length \( L \propto \) total energy = \( E \)
signal \( S \) (mainly due to low-energy particles) \( \propto L \propto E \)
→ linearity

fluctuations:

a 40 GeV shower equivalent to \( 2 \times 20 \) GeV showers
→ independent fluctuations
→ \( \sigma(E) \propto \sqrt{E} \)

stochastic term:

\[ \sigma(E)/E = a/\sqrt{E} \rightarrow \text{improves as } E^{-1/2} \]
energy resolution

quartz: better UV light transmittance
**sampling calorimeters**

usually sandwich of active (e.g. scintillator plates) and passive elements (e.g. lead plates)

→ impact on resolution?

sampling fraction: fraction of energy lost in the active medium (by a minimum ionising particle)
(rough) rule of thumb: \( a_{\text{samp}} = 2.7\% \sqrt{d/f_{\text{samp}}} \)

\( d \ [ \text{mm} ] = \text{thickness of each active layer} \)
1) homogeneous: 100% of shower track sampled in active medium

→ resolution $\sigma/E \sim O(1\%)/\sqrt{E}\,(\text{GeV})$

2) sampling: only part ($<\sim 5\%$) of track sampled in active medium

→ resolution $\sigma/E \sim O(10\%)/\sqrt{E}\,(\text{GeV})$

* “typical” values for high-energy physics
### real em calorimeters

<table>
<thead>
<tr>
<th>Technology (Experiment)</th>
<th>Depth</th>
<th>Energy resolution</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaI(Tl) (Crystal Ball)</td>
<td>20(X_0)</td>
<td>2.7%/(E^{1/4})</td>
<td>1983</td>
</tr>
<tr>
<td>Bi(_4)Ge(_3)O(_12) (BGO) (L3)</td>
<td>22(X_0)</td>
<td>2%/(\sqrt{E}) (\oplus) 0.7%</td>
<td>1993</td>
</tr>
<tr>
<td>CsI (KTeV)</td>
<td>27(X_0)</td>
<td>2%/(\sqrt{E}) (\oplus) 0.45%</td>
<td>1996</td>
</tr>
<tr>
<td>CsI(Tl) (BaBar)</td>
<td>16–18(X_0)</td>
<td>2.3%/(E^{1/4}) (\oplus) 1.4%</td>
<td>1999</td>
</tr>
<tr>
<td>CsI(Tl) (BELLE)</td>
<td>16(X_0)</td>
<td>1.7% for (E_\gamma &gt; 3.5) GeV</td>
<td>1998</td>
</tr>
<tr>
<td>PbWO(_4) (PWO) (CMS)</td>
<td>25(X_0)</td>
<td>3%/(\sqrt{E}) (\oplus) 0.5% (\oplus) 0.2/(E)</td>
<td>1997</td>
</tr>
<tr>
<td>Lead glass (OPAL)</td>
<td>20.5(X_0)</td>
<td>5%/(\sqrt{E})</td>
<td>1990</td>
</tr>
<tr>
<td>Liquid Kr (NA48)</td>
<td>27(X_0)</td>
<td>3.2%/(\sqrt{E}) (\oplus) 0.42% (\oplus) 0.09/(E)</td>
<td>1998</td>
</tr>
<tr>
<td>Scintillator/depleted U (ZEUS)</td>
<td>20–30(X_0)</td>
<td>18%/(\sqrt{E})</td>
<td>1988</td>
</tr>
<tr>
<td>Scintillator/Pb (CDF)</td>
<td>18(X_0)</td>
<td>13.5%/(\sqrt{E})</td>
<td>1988</td>
</tr>
<tr>
<td>Scintillator fiber/Pb spaghetti (KLOE)</td>
<td>15(X_0)</td>
<td>5.7%/(\sqrt{E}) (\oplus) 0.6%</td>
<td>1995</td>
</tr>
<tr>
<td>Liquid Ar/Pb (NA31)</td>
<td>27(X_0)</td>
<td>7.5%/(\sqrt{E}) (\oplus) 0.5% (\oplus) 0.1/(E)</td>
<td>1988</td>
</tr>
<tr>
<td>Liquid Ar/Pb (SLD)</td>
<td>21(X_0)</td>
<td>8%/(\sqrt{E})</td>
<td>1993</td>
</tr>
<tr>
<td>Liquid Ar/Pb (H1)</td>
<td>20–30(X_0)</td>
<td>12%/(\sqrt{E}) (\oplus) 1%</td>
<td>1998</td>
</tr>
<tr>
<td>Liquid Ar/depl. U (DØ)</td>
<td>20.5(X_0)</td>
<td>16%/(\sqrt{E}) (\oplus) 0.3% (\oplus) 0.3/(E)</td>
<td>1993</td>
</tr>
<tr>
<td>Liquid Ar/Pb accordion (ATLAS)</td>
<td>25(X_0)</td>
<td>10%/(\sqrt{E}) (\oplus) 0.4% (\oplus) 0.3/(E)</td>
<td>1996</td>
</tr>
</tbody>
</table>
hadronic calorimetry

$\pi^0, \eta^0$ production → hadronic showers develop 2 main components:

h component: p, n, $\pi^\pm$, nuclear fission, … delayed photons, …

dimension scale: $\lambda_1 \sim 35 \text{ g/cm}^2 \cdot A^{1/3}$
hadronic shower development

on average equivalent to em showers with $X_0$ and $R_M$ replaced by $\lambda_1$ (nuclear interaction length) … but:

single showers by 170 GeV pions

… much much larger event-by-event fluctuations!
→ a factor $> \sim 10$ in $\lambda_1/X_0$ ratio
hadronic shower components

- **Electromagnetic component**
  - electrons, photons
  - neutral pions → 2 γ

- **Hadronic (non-em) component**
  - charged hadrons $\pi^\pm, K^\pm$ (20%)
  - nuclear fragments, p (25%)
  - neutrons, soft γ’s (15%)
  - break-up of nuclei (“invisible”) (40%)

many components w/ large fluctuations in relative yield

1. large non-gaussian fluctuations in energy sharing em/non-em
2. increase of em component with energy
3. large, non-gaussian fluctuations in “invisible” energy losses
**electromagnetic fraction** $f_{em}$

energy fraction carried by $\pi^0$ (mainly) and $\eta^0$

$f_{em}$, on average, *large and energy dependent*
fluctuations in $f_{em}$ *large and non-poissonian*

$$\langle f_{em} \rangle = 1 - \left( \frac{E}{E_0} \right)^{(k-1)}$$

$E_0 =$ average energy to produce a $\pi^0$
(k-1) related to average multiplicity

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Parameterization:

$$f_{em} = 1 - \left[ \frac{E}{E_0} \right]^{(k-1)}$$

- $Cu$ ($k = 0.82, E_0 = 0.7 \text{ GeV}$)
- $Pb$ ($k = 0.82, E_0 = 1.3 \text{ GeV}$)
- NIM A316 (1992) 184
- NIM A399 (1997) 202
\[ f_{em} \text{ fluctuations} \]

\[ f = \frac{c - s(C/S)}{(C/S)(1 - s) - (1 - c)} \]

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**DREAM: Effect of event selection based on \( f_{em} \)**

- **Entries:** 78198
- **Mean:** 66.1
- **RMS:** 12.4

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From: 
NIM A537 (2005) 537
In nuclear reactions energy is lost (binding energy) to free protons and neutrons. It can’t provide any measurable signal (invisible energy) and accounts on average for ~ 30-40% of non-em shower energy. Large event-by-event fluctuations limit resolution.

Correlation between invisible energy and kinetic energy carried by released nucleons.

Evaporation nucleons: soft spectrum, mostly neutrons (2-3 MeV).

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Fig. 4.43. The nuclear binding energy lost in spallation reactions induced by 1 GeV protons on $^{238}$U nuclei (a), and the number of neutrons produced in such reactions (b). From [Wig 87].
Measurement of the kinetic energy of neutrons - correlated to nuclear binding energy loss (invisible energy) - from signal time structure (DREAM)

**Signal time structure**

- **200 GeV “jets”**
- Čerenkov
- Scintillator

\[ \tau = 20 \text{ ns} \]

**f_n anti-correlated to f_{em}**

- Neutron fraction in scintillator signal

**Probing the tot. signal distribution with f_n**

- Number of entries per bin
- Total Čerenkov signal (arbitrary units)

- All data
- \(0.06 < f_n < 0.065\)
- \(0.07 < f_n < 0.075\)
- \(0.08 < f_n < 0.085\)

**no tail in em showers**
Response:

\textit{detected signal per unit energy deposit}

e.g. number of scintillating (or Cherenkov) p.e. / deposited GeV

Hadronic showers:
- em component $\rightarrow$ response e
- hadronic component $\rightarrow$ response h

\textit{what about the relative ratio e/h ?}
detector response to hadronic showers

\[ e \neq h \]

e.g. (right plot):
only \( 1/1.8 \approx 56\% \) of non-\( \pi^0 \) energy accounted by signal

**Note:**
e/h ratio: detector characteristic
typically, \( \sim 2 \) for crystals, in range 1-1.8 for sampling calorimeters

Nevertheless:

1) \( e/\pi \) depends on energy \( (f_{em} \text{ depends on } E \text{ and shower “age”}) \)
2) \( f_{em} \) different for \( \pi, K, p \rightarrow \) response depends of particle type
compensation

e/h = 1 → compensating calorimeter

1) increase h → boost hadron response
e.g. by adding hydrogen or by using Uranium, both acting as “neutron converters” → large integration volume and time

2) decrease e → decrease em sampling fraction (i.e. em performance) → tune active / passive material ratio
**compensation pros & cons**

- **not a guarantee for high resolution**
  - fluctuations in $f_{\text{em}}$ are eliminated, but others may be very large

- **has drawbacks**
  - high-Z absorber required $\rightarrow$ small e/mip $\rightarrow$ non linearity @ low energy
  - low sampling fraction required $\rightarrow$ em resolution limited
  - relies on neutrons $\rightarrow$ integration over large volume and time
    - *SPACAL 30%$/\sqrt{E}$ needed ~15 tonnes and ~50 ns*

- high-res em and high-res hadron calorimetry mutually exclusive:
  - **good jet energy resolution $\Rightarrow$ compensation**
    - $\Rightarrow$ small sampling fraction ($\sim$3%) $\Rightarrow$ poor em resolution
  - **good em resolution $\Rightarrow$ high sampling fraction (100% crystals, 20% LAr)**
    - $\Rightarrow$ large non compensation $\Rightarrow$ poor jet resolution
**real hadronic calorimeters**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Detector</th>
<th>Absorber material</th>
<th>e/h</th>
<th>Energy resolution (E in GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UA1 C-Modul</td>
<td>Scintillator</td>
<td>Fe</td>
<td>$\approx 1.4$</td>
<td>$80%/\sqrt{E}$</td>
</tr>
<tr>
<td>ZEUS</td>
<td>Scintillator</td>
<td>Pb</td>
<td>$\approx 1.0$</td>
<td>$34%/\sqrt{E}$</td>
</tr>
<tr>
<td>WA78</td>
<td>Scintillator</td>
<td>U</td>
<td>0.8</td>
<td>$52%/\sqrt{E} \pm 2.6%^*$</td>
</tr>
<tr>
<td>D0</td>
<td>liquid Ar</td>
<td>U</td>
<td>1.11</td>
<td>$48%/\sqrt{E} \pm 5%^*$</td>
</tr>
<tr>
<td>H1</td>
<td>liquid Ar</td>
<td>Pb/Cu</td>
<td>$\leq 1.025^*$</td>
<td>$45%/\sqrt{E} \pm 1.6%$</td>
</tr>
<tr>
<td>CMS</td>
<td>Scintillator</td>
<td>Brass (70% Cu / 30% Zn)</td>
<td>$\approx 1$</td>
<td>$100%/\sqrt{E} \pm 5%$</td>
</tr>
<tr>
<td>ATLAS (Barrel)</td>
<td>Scintillator</td>
<td>Fe</td>
<td>$\approx 1$</td>
<td>$50%/\sqrt{E} \pm 3%$</td>
</tr>
<tr>
<td>ATLAS (Endcap)</td>
<td>liquid Ar</td>
<td>Brass</td>
<td>$\approx 1$</td>
<td>$60%/\sqrt{E} \pm 3%$</td>
</tr>
</tbody>
</table>

* after software compensation
What?

Don’t spoil em resolution to get $e/h = 1$ (i.e. keep $e/h > 1$) \textit{BUT} measure $f_{\text{em}}$ event-by-event

$\Rightarrow$ \textit{correct energy measurements for $f_{\text{em}}$ fluctuations}

How?

Exploit the fact that (e/h) values for scintillation light (S) and Čerenkov light (Č) production processes are (very) different

Why?

\textit{Charged hadrons contribute to S but very marginally to Č}
working principles

\[
S = E \cdot \left[ f_{\text{em}} + (h/e)_s \cdot (1 - f_{\text{em}}) \right] = E \cdot \left[ f_{\text{em}} + s \cdot (1 - f_{\text{em}}) \right]
\]

\[
C = E \cdot \left[ f_{\text{em}} + (h/e)_c \cdot (1 - f_{\text{em}}) \right] = E \cdot \left[ f_{\text{em}} + c \cdot (1 - f_{\text{em}}) \right]
\]

\[(h/e)_s = s, \quad (h/e)_c = c \rightarrow \text{detector-specific parameters}\]

Both \( f_{\text{em}} \) and \( E \) can be reconstructed:

\[
f = \frac{c - s(C/S)}{(C/S)(1 - s) - (1 - c)}
\]

\[
E = \frac{S - \chi C}{1 - \chi}
\]

where:

\[
\chi = \frac{1 - s}{1 - c} = \frac{E - S}{E - C}
\]

\( \chi \) can be evaluated from testbeam data
to be continued ...